EXAM P/1 PROBABILITY

New York University Actuarial Society

Wednesday, March 22nd, 2017

EXAM TOPICS

1. General Probability
2. Random Variables with Univariate Distribution
3. Random Variables with Multivariate Probability Distribution

Prerequisites: Calculus(Double integrals), Basic insurance concepts

GENERAL PROBABILITY (10-20%)

- Set theory
- Addition/Multiplication rules
- Mutually exclusive events
- Independence of events
- Combinatorial probability
- Conditional probability
- Bayes Theorem/Law of Total Probability

BASIC PROBABILITY CONCEPTS

Probability space - all possible outcomes

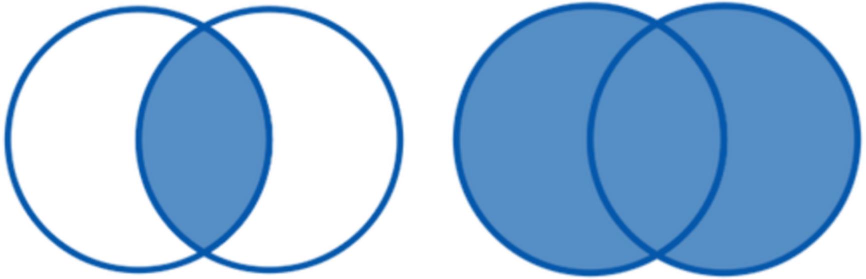
Event - a subset of a probability space where probability can be calculated

Union $A \cup B$ - A union of two events is an event which combines all of their elements

Intersection $A \cap B$ - An intersection of two events is an event whose elements belong to both events

Complement A/B - the event where A occurs but not B

$P(A \cup B)=P(A)+P(B)-P(A \cap B)$



SIMPLE EXAMPLE

Flip a coin three times where order doesn't matter

Probability space: {HHH, HHT, TTH, TTT}

An event, A: At least 1 heads {HHH, HHT, TTH}

$P(A) = \frac{3}{4}$

An event, B: At least 1 heads and 1 tails {HHT, TTH}

$P(B) = \frac{1}{2}$

$P(A \cup B) = ?$
$P(A \cap B) = ?$

CONDITIONAL PROBABILITY LAW OF TOTAL PROBABILITY

Probability of A given event B: $P(A|B)= P(A\cap B)/P(B)$

Why?

To find the probability we usually what to find the occurrences of an event and divide it by the entire probability space.

We know B already happened. So all the events of our probability space are P(B). The event where A happens and B happened is simply $P(A\cap B)$.

$P(A\cap B)=P(A|B)*P(B)$

Law of Total Probability: $P(A)= P(A\cap B)+P(A\cap B') = P(A|B)*P(B)+P(A|B')*P(B')$

EXAMPLE

A = You pass Exam P
B = You go to NYU ActSoc Meetings

$P(A) = .5$
$P(B) = .6$
$P(A|B') = .2$

Find the probability that you pass Exam P given you go to NYU ActSoc Meetings

BAYES' THEOREM (IMPORTANT!)

P(A|B)= P(A∩B)/P(B) = (P(B|A)*P(A))/(P(B|A)*P(A)+P(B|A')*P(A'))

Why?

P(A∩B) = P(B|A)*P(A) (Definition of conditional probability)

P(B) = P(B|A)*P(A)+P(B|A')*P(A') (Law of total probability)

Extends to many events for which E is partitioned (mutually exclusive events)

$$\Pr(E_i|A) = \frac{\Pr(A|E_i)\cdot\Pr(E_i)}{\sum_{j=1}^{n}\Pr(A|E_j)\cdot\Pr(E_j)} = \frac{\Pr(A|E_i)\cdot\Pr(E_i)}{\Pr(A|E_1)\cdot\Pr(E_1)+\ldots+\Pr(A|E_n)\cdot\Pr(E_n)}.$$

BAYES THEOREM EXAMPLE:

Exercise 1.5. May 2003 Course 1 Examination, Problem No. 31, also P Sample Exam Questions, Problem No. 22, also Dr. Ostaszewski's online exercise posted March 1, 2008
A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

A. 0.20 B. 0.25 C. 0.35 D. 0.42 E. 0.57

INDEPENDENCE VS. MUTUALLY EXCLUSIVE

Independent events (A does not depend on B and vice versa):
$P(A \cap B) = P(A) * P(B)$
$P(A|B) = P(A)$
e.g. flipping a coin, rolling a dice

Mutually Exclusive (Cannot happen at the same time):
$P(A \cap B) = 0$
$P(A|B) = 0$
e.g. pass/fail, enrolled in courses, age

COMBINATORIAL PROBABILITY

If we have n elements, there are n ways of choosing the first element, n-1 ways of choosing the second element, n-2 ways of choosing the third, etc.

That means there are n! ways of ordering the n elements. These are called **permutations.**

52! ways a deck of cards can be shuffled - 8*10^67.

If we want to order them by k elements there are n ways of choosing the first element, n-k ways of choosing the second element, n-2k ways of choosing the third, etc.

That is, n!/(n-k)!

If order doesn't matter, we have **combinations.** Since k elements can be ordered in k! different ways, we divide the permutations by k!. We have n!/((n-k)!*k!). This is read as n choose k.

COMBINATORICS EXAMPLE:

Exercise 1.11. February 1996 Course 110 Examination, Problem No. 7, also Dr. Ostaszewski's online exercise posted June 26, 2010

A class contains 8 boys and 7 girls. The teacher selects 3 of the children at random and without replacement. Calculate the probability that the number of boys selected exceeds the number of girls selected.

A. $\frac{512}{3375}$ B. $\frac{28}{65}$ C. $\frac{8}{15}$ D. $\frac{1856}{3375}$ E. $\frac{36}{65}$

STUDY RESOURCES

Free

- SOA Sample Questions
- SOA Sample Exam
- Numerous practice exams can be found online

Free-ish

- ASM Manual
- ACTEX Manual

Paid

- Coaching Actuaries
- The Infinite Actuary

STUDY TIPS

- SOA recommends 100 hours of studying per hour of exam
- Create a study schedule
- Read a manual to learn the material
  - Skim, jot notes down, try to understand the intuition
  - Don’t spend too much time on this
- Do practice problems
  - For Exam P do the SOA online questions over and over again, there are 250 of them
  - If you need it, buy a Coaching Actuaries subscription, if your Earned Level reaches 7 you have a 90% chance of passing
- Find out what works for you! The exam problems aren’t extremely difficult but being able to do them quickly and accurately is what is problematic.

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