Raj and Sarah

Intro to Probability and Statistics

Upcoming Events

Mentorship

Speed Dating 9/18

Kickoff and Networking Mingle 9/25

Actuarial Career Summit

September 27th KMEC 550 1400-1700

Business Formal

Opportunities

- D E Shaw (Internship and Full-time positions)
- Third Point RE (Fall Part-time + 2020 Summer)
  - Sophomores or Juniors
  - Resume and cover letter to albert.zhou@thirdpointre.com
- 2019 ASNY Career Fair
  - Registration through Sept. 13 is free!
  - Held on Sunday, October 6 from 10 AM - 6PM

What is Probability?

- The extent to which an event is likely to occur, measured by the ratio of the favorable cases to the whole number of cases possible.

$$
\begin{aligned}
&(1)\ P(A) \geq 0 \text{ for all } A \subset S \\
&(2)\ P(S) = 1 \\
&(3)\ \text{If } A \cap B = \emptyset, \\
&\quad\ \text{then } P(A \cup B) = P(A) + P(B)
\end{aligned}
$$

Basic Relationships

De Morgan's Law

$$\Pr[(A \cup B)'] = \Pr(A' \cap B')$$
$$\Pr[(A \cap B)'] = \Pr(A' \cup B')$$

Law of Total Probability

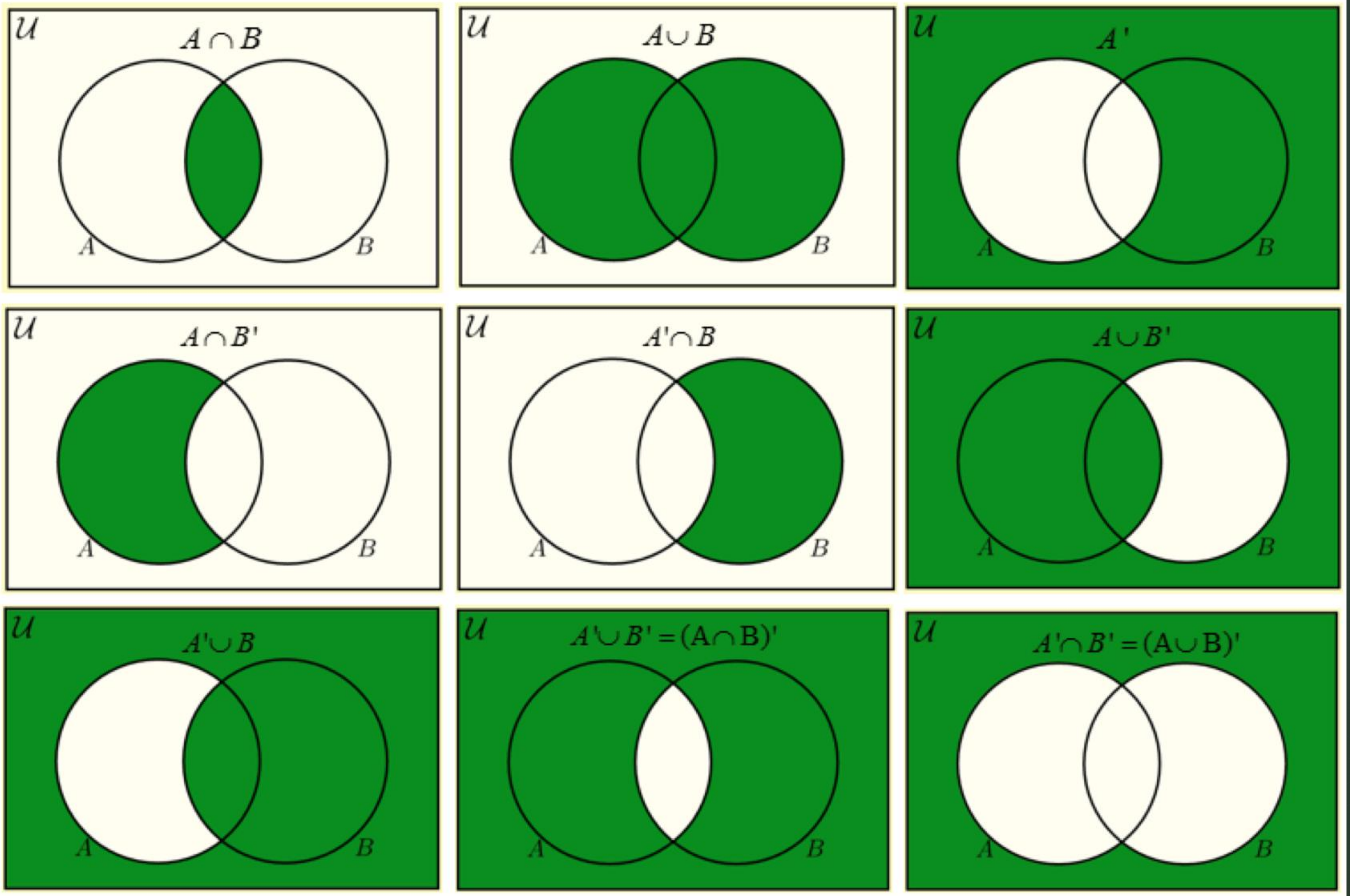
$$\Pr(B) = \sum_{i=1}^{n} \Pr(B \cap A_i)$$

Independence

$$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$$

Basic Probability Relationships

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$\begin{aligned}\Pr(A \cup B \cup C) = {} & \Pr(A) + \Pr(B) + \Pr(C) \\ & - \Pr(A \cap B) - \Pr(B \cap C) \\ & - \Pr(A \cap C) + \Pr(A \cap B \cap C)\end{aligned}$$
$$\Pr(A') = 1 - \Pr(A)$$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A ∩ B
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A' ∪ B' = (A ∩ B)'
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U
A' ∩ B' = (A ∪ B)'
A
B

PDF/PMF/CDF ???

- What are thoseeeeeee

Discrete and Continuous Events

- Discrete examples:
  - Probability of getting 7 heads in 10 flips of a coin
  - Probability of rolling an even number on a six-sided die
- Continuous examples:
  - Probability of a person being over 6 feet tall
  - Time it takes until a radioactive particle decays

Distributions

- Discrete
  - Uniform, Binomial, Poisson, Geometric
- Continuous
  - Uniform, Normal, Exponential

Conditional Probability & Bayes' Theorem

- Conditional probability:
  - New universe space defined due to a certain event occurring
  - $P(A|B) = \frac{P(A \cap B)}{P(B)}$; with P(B) being the new universe
- Independence: $P(A) = P(A|B)$
- Bayes' Theorem:

Bayes' Theorem

$$\Pr(A_i|B) = \frac{\Pr(B|A_i) \cdot \Pr(A_i)}{\sum_{k=1}^{m} \Pr(B|A_k) \cdot \Pr(A_k)}$$

Bayes' Theorem Example

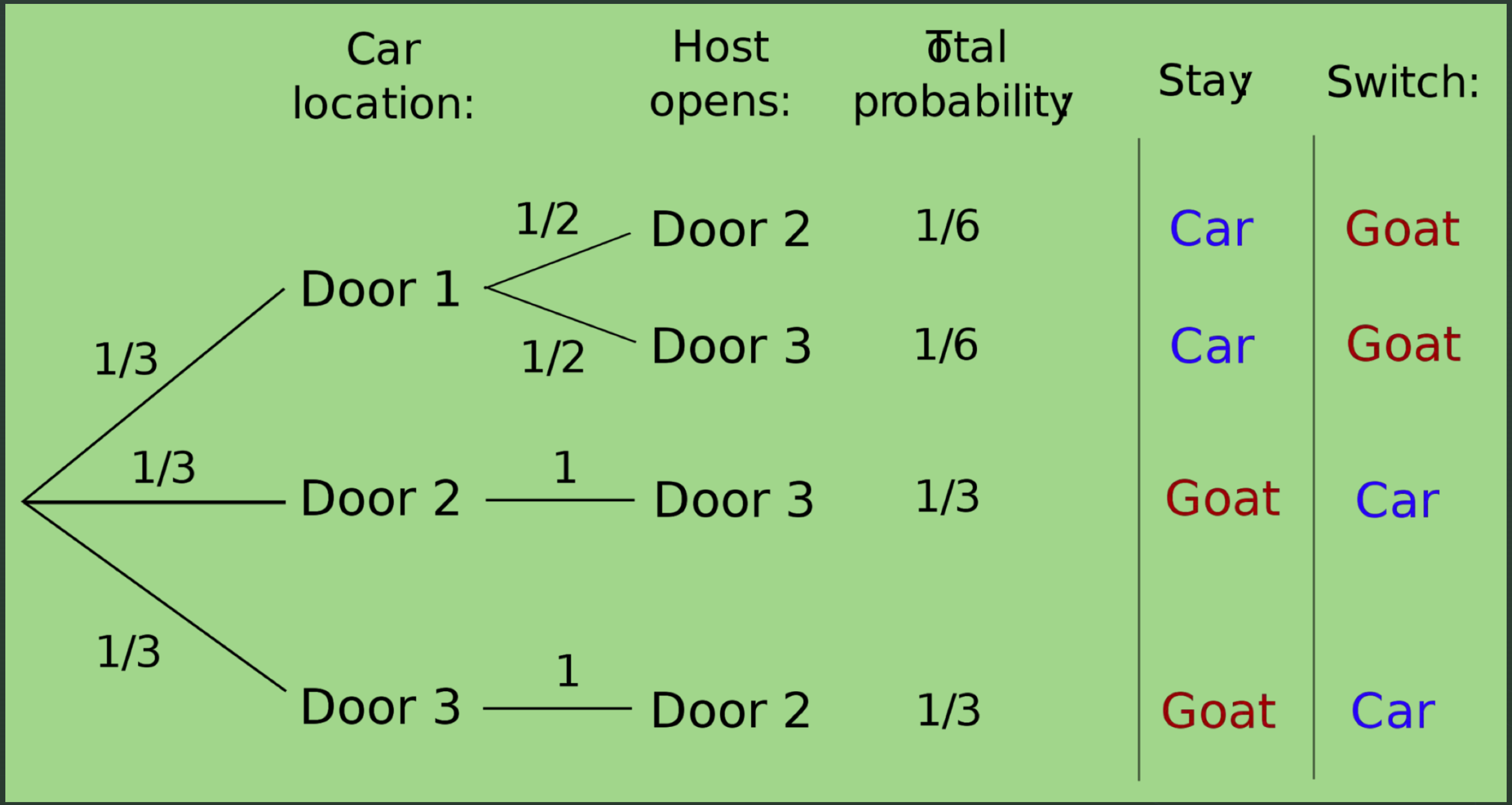
- Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?

Another Bayes' Theorem Example!

- The Monty Hall Problem:

  Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, *who knows what's behind the doors*, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

A Solution


Car location:
Host opens:
Total probability
Stay
Switch:
1/3
1/3
1/3
Door 1
Door 2
Door 3
1/2
1/2
1
1
Door 2
Door 3
Door 3
Door 2
1/6
1/6
1/3
1/3
Car
Car
Goat
Goat
Goat
Goat
Car
Car