Exam P: Probability!

Upcoming Events:

- **Interview + Resume Workshop**
  - Wed 2/26, 12:30-1:45 PM, UC-19

- **Intro to Risk Management w/LT**
  - Fri 2/28, 5-6 PM, UC-19

Opportunities:

- **Sompo International** - Summer Internship Opportunity
  - Email your resume to: echiang@sompo-intl.com

- **Horizon Blue Cross Blue Shield** - Summer Internship Opportunity
  - Email your resume to: Jedwin_Celestino@horizonblue.com and CC: Rhaea_Guieb@horizonblue.com
  - Must have 1 exam passed
  - Due February 21st

Format:

- 3 Hours
- 30 Multiple Choice Questions

Exam Topics:

- General Probability (10-17%)
- Univariate Random Variables (40-47%)
- Multivariate Random Variables (40-47%)

Conditional Probability

- The probability of one event given that another has occurred

$$P(\mathrm{A} \mid \mathrm{B}) = \frac{P(\mathrm{A\ and\ B})}{P(\mathrm{B})}$$

- Bayes Theorem!!

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{\sum_{\mathrm{i}=1}^{\mathrm{n}}[\mathrm{P}(\mathrm{B}|\mathrm{A_i})\mathrm{P}(\mathrm{A_i})}$$

Conditional Probability Example:

Arjun hands in his homework on time 40% of the time. Kayla hands in her homework on time 85% of the time. Both Arjun and Kayla hand in their homework on time 30% of the time. Find the probability that Kayla hands in her homework on time, given Arjun does not hand in his homework on time.

92%

Example:

A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died during the five-year period.

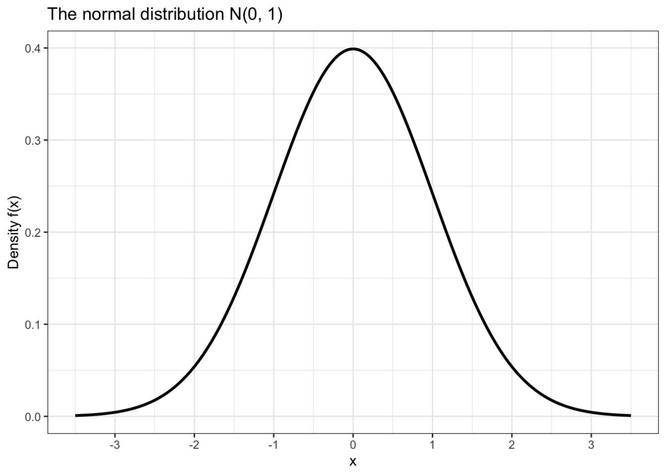
Calculate the probability that the participant was a heavy smoker.

(A) 0.20
(B) 0.25
(C) 0.35
(D) 0.42
(E) 0.57

CDF's

Cumulative Distribution Functions: probability that will take a value less than or equal to

$$F(x) = P(a \leq x \leq b) = \int_a^b f(x)dx \geq 0$$



Example :

The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x), & 0 < x < 20 \\ 0, & \text{otherwise.} \end{cases}$$

Given that a fire loss exceeds 8, calculate the probability that it exceeds 16.

(A) 1/25
(B) 1/9
(C) 1/8
(D) 1/3
(E) 3/7

Finding Expected Values

Find and use the expected values of a distribution for further analysis

Sum of (each possible outcome * the probability of that outcome)

Some Properties of E[X]

- E[X + Y] = E[X] + E[Y]
- E[n] = n (when n is a constant)

$$E[X] = \sum_i x_i f(x_i)$$

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx$$

Example:

Joonyoung is NYU's best swimmer. Every time she gets first place in her race, she is rewarded with $100. Calculate the expected value of her total reward over the course of one swimming season using the following table:

Number of First place finishes	0	1	2	3	4
Probability	.15	.15	.25	.35	.1

$210!!

Example:

A dental insurance company pays 100% of the cost of fillings and 70% of the cost of root canals. Fillings and root canals cost 50 and 500 each, respectively.

The tables below show the probability distributions of the annual number of fillings and annual number of root canals for each of the company's policyholders.

# of Fillings	0	1	2	3
Probability	0.60	0.20	0.15	0.05

# of Root Canals	0	1
Probability	0.80	0.20

Calculate the expected annual payment per policyholder for fillings and root canals.

(A) 90.00
(B) 102.50
(C) 132.50
(D) 250.00
(E) 400.00

Distributions

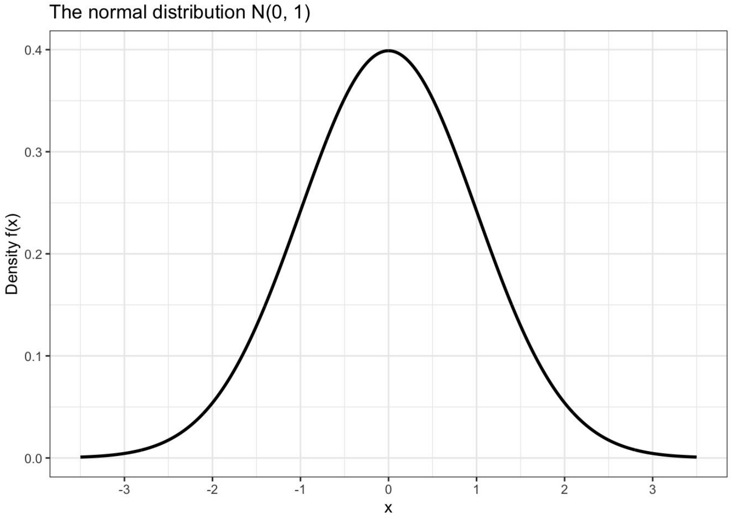
Certain distributions can accurately model different aspects of an insurance policy

- For example: loss amounts, number of claims, profits,

Important Aspects of all Distributions

- Mean = Average = Expected Value (E[X])
- Variance (σ^2) = measure of spread
- Standard Deviation = σ
- PDF
- CDF

Normal Distribution



Bell Curve

Symmetrical Distribution

Z-score: measures how many standard deviations a value, x, is from the mean

$$z = \frac{x - \mu}{\sigma}$$

Example:

An insurance company's annual profit is normally distributed with mean 100 and variance 400.

Let Z be normally distributed with mean 0 and variance 1 and let F be the cumulative distribution function of Z.

Determine the probability that the company's profit in a year is at most 60, given that the profit in the year is positive.

(A) $1 - F(2)$

(B) $F(2)/F(5)$

(C) $[1 - F(2)]/F(5)$

(D) $[F(0.25) - F(0.1)]/F(0.25)$

(E) $[F(5) - F(2)]/F(5)$